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Optimal energy management in a dual-storage fuel-cell hybrid vehicle using multi-dimensional dynamic programming

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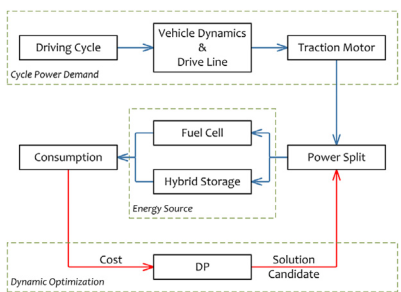
HIGHLIGHTS

- Investigation of the model-based optimal energy management of a fuel-cell hybrid vehicle.
- Vehicle equipped with battery and ultracapacitor hybrid storage system.
- Development of multi-dimensional dynamic programming code for 2-DOF energy management problem.
- Determination of maximum potential of using hybrid storage in reducing fuel consumption.
- Hybrid storage improved fuel consumption especially in acceleration and braking.

GRAPHICAL ABSTRACT



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ABSTRACT

Hybrid storage systems consisting of battery and ultra-capacitor have recently emerged as an alternative to the conventional single buffer layout in hybrid vehicles. Their high power and energy density could improve the performance indices of the vehicle, provided that an optimal energy management strategy is employed that could handle systems with multiple degrees of freedom (DOF). The majority of existing energy management strategies is limited to a single DOF and the small body of work on multi-DOF systems is mainly heuristic-based.

We propose an optimal solution to the energy management problem in fuel-cell hybrid vehicles with dual storage buffer for fuel economy in a standard driving cycle using multi-dimensional dynamic programming (MDDP). An efficient MDDP code is developed using MATLAB™'s vectorization feature that helps reduce the inherently high computational cost of MDDP. Results of multiple simulated experiments are presented to demonstrate the applicability and performance of the proposed strategy. A comparison is also made between a single and a double buffer fuel-cell hybrid vehicle in various driving cycles to determine the maximum reduction in fuel consumption that can be achieved by the addition of an ultra-capacitor.

1. Introduction

One of the achievements of modern engineering is improvement in the performance of existing systems by replacing the internal components and updating subsystem technology with the help of computer software and simulation packages. This branch of engineering is associated with design study and optimization and its main challenge is prediction of enhancement in system attribute arising from changes in system parameters.

In the face of a constant stream of criticisms, hybrid vehicles (HVs) enjoy a growing popularity within the engineering

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community and among consumers alike. The introduction of newer powertrain layouts has considerably improved the performance of HVs and enabled them to meet the increasingly strict environmental regulations and economic considerations. The inevitable transition from conventional powertrains to those that rely less on fossil fuels justifies the extensive research on hybridization technology that is underway both in industry and in academia.

The use of auxiliary energy sources in HVs leads to additional degrees of freedom in power distribution. This means that the performance of a hybrid vehicle largely depends on the efficiency of the energy management strategy employed. Devising an efficient energy management strategy involves the solution of a dynamic optimization problem wherein such cumulative quantities as fuel consumption constitute the cost function and the power distribution profile among various sources (as a function of time) forms the design variable set. That is, the optimal power management can be formulated as finite-horizon nonlinear optimal control problem.

Of the various reported techniques to solve the above control problem, dynamic programming (DP) is one of the most popular. It guarantees the optimality of the solution found for a given tolerance; its mathematical foundations are straightforward; and it could be easily implemented in a concise computer code. But dynamic programming has some shortcomings, too. It requires a variety of algorithms to tackle the numerical problems and is case-dependent. But the main drawback is well known Bellman's the "curse of dimensionality", the computational cost increases exponentially with the number of variables (state and control) [1]. Also the interpolation problem arising from a misplaced state trajectory and grid points poses a major challenge.

Using the code-vectorization feature of MATLAB to avoid iterative loop structures, a generic and computationally efficient algorithm is presented here for MDDP problems. A general approach to the optimal energy management is adopted throughout the article, and a fuel-cell hybrid vehicle (FCHV) is considered as a case study. Many researchers declared fuel cell technology as a high-chance candidate to replace combustion engines in cars in nearly future [2].

Up to now, few papers have evaluated the fuel economy potential of the fast ultra-capacitor and traditional battery combination. These few works only focused on heuristic controllers. The main contribution of this paper is to report the solution of optimal two DOF energy management problem and investigating the applicability of DP to multi-dimensional problems. By developing a new solver based on vectorized multi-dimensional dynamic programming, the possibility to solve more problems in this field afforded. Also by assembling DP kernel and hybrid vehicle simulator, an integrated user interface presented for interested researchers.

2. Structure of the article

The overall objective of this study is to evaluate the effect of a hybrid storage system on the performance of a Hybrid Electric Vehicle. Generally, the simultaneous use of batteries (high-specific-energy) and ultra-capacitors (high-specific-power) leads to performance improvement of the HEV [3], but the implementation costs are unaffordably high. For an accurate feasibility assessment of using a hybrid storage in practical applications, the tradeoff between the advantages (including reduced fuel consumption, improved acceleration and braking, and reduced main storage capacity) and disadvantages (including implementation and maintenance considerations, increased hardware complexity, and additional cost of interface components) of the hybrid layout should be carefully studied.

In this study, the optimal power distribution profile to achieve minimum fuel consumption in a fuel cell hybrid vehicle with hybrid storage is determined in a standard driving cycle. The minimum fuel consumption is a comprehensive and practical indicator for evaluating the hybrid storage system. To use the maximum potential of system in reduction of fuel consumption, an independent degree of freedom in power distribution is considered for each storage element.

To achieve this, first a standard driving cycle profile is considered and fed to an inverse model of vehicle dynamic and electric motor to calculate the power demand from propulsion system. Then a simple model of the powertrain has been developed, which can be used for applications such as control analysis and design and parametric studies. This model has two state variables (battery and ultra-capacitor state of charge), two control inputs (fuel cell and ultra-capacitor power) and external disturbance as a function of time (driving cycle power request from the propulsion system). Now the energy management problem is formulated as a finite-horizon nonlinear optimal control with bounded states and control variables. The optimal control problem is solved using multi-dimensional, vectorized dynamic programming implemented in MATLAB. The cost functional includes charge sustenance (equal energy level of storage at the beginning and end of the cycle) and fuel consumption. This structure simulated in variety of driving cycles and the performance of fuel cell hybrid vehicle with simple and dual storage is compared, so the maximum potential of the fuel economy by adding fast storage (ultra-capacitor) is assessed.

This research provides the foundation for model-based studies of fuel cell hybrid vehicles which has wide applications and covers a range of issues such as sizing and optimization of components and control strategies development and evaluation. By using MATLAB, a graphical user interface was developed that automate problem analysis. Based on developed tool, the possibility of using hybrid storage in fuel cell hybrid vehicle was studied.

3. Literature survey

This section reviews the major studies in the field of energy management in hybrid vehicles and in particular deals with the optimal energy management in fuel cell hybrid vehicles based on dynamic programming. This review focused on vacuity existing in the field of multi-dimensional optimal control in energy management problems. See Ref. [4] for supplementary studies.

3.1. Energy management

The term "energy management" in the literature refers to how road load distributes between different energy sources while satisfying design requirements (fuel consumption, pollution). There are two main categories, heuristic and optimal, proposed to solve energy management problems [4]. Heuristic methods used for real-time applications, and despite the high sensitivity to rule-base tuning, they offer acceptable performance. On the contrary, the optimal approaches are based on the minimizing a cost function. These methods are typically model-based and therefore scalable and extensible. Computational burden is much higher with the offline optimal methods compared with the online heuristic methods.

The optimal strategies in energy management are divided into instantaneous optimization and finite-horizon optimization and widely used in design of supervisory control [5]. Instantaneous methods provide power flow for real-time applications based on local criteria. Finite-horizon methods include dynamic indexes such as energy consumption during a priori known driving cycle and despite offering the global optimal solution, they cannot implemented in real-time applications, because of need to non-causal knowledge of the automotive motion situation in the

future. Because of the time and state dependent parameters and nonlinear effects, this problem belongs to finite-horizon nonlinear optimal control with constrained state and control.

Guzzella et al. [4] comprehensively studied hybrid vehicles and energy management. They overviewed various methods of hybrid vehicle control that includes two main topics, "Optimal Energy Management" and "Real-Time Implementation" and alluded to the research background, the present work and trends of future research. Perez studies the energy management problem in a global coordinate and independent of vehicle layout and the formulation presented is applicable to various configurations of vehicle [6].

3.2. Energy sources

Typically the structure of the hybrid vehicle powertrain consists of two main elements: the primary power source (one-way energy conversion unit) and the secondary power source (bi-directional energy buffer). The storage device provides the ability to recovering brake energy, increase of power in acceleration mode and avoiding engine work in low-efficiency region. Various tools are used as energy storage device that two important parameters, the specific power and energy are used to characterize them. An ideal storage should have high energy storage and fast energy release capabilities.

The battery plays role of storage in most of hybrid vehicles. Meanwhile the main problem with conventional batteries is high sensitivity to the rate of power which causes limitation in energy storage and retrieval performance and therefore they cannot offer good performance in acceleration and deceleration circumstances. Other challenge in batteries is limitations due to working cycles, which highly affected their life and durability. Ultra-capacitor located on the opposite side and can work in high power and frequency situation and infinite number of working cycle. The use of ultra-capacitor is not bug-free and it imposed electronic interface elements to the system, thereby increasing the overall cost of system, increased complexity and special considerations in maintenance.

Combining two or more storage sources of energy is one way to achieve the ideal Hybrid Storage System so they Undermine shortcomings and reinforce advantages of each other [2]. One of the common patterns of idealized storage that generally used in hybrid electric vehicles is combination of a fast buffer (ultra-capacitor) to aid main buffer (battery) and cover its deficiency. The battery and ultra-capacitor combined system has high performance and low weight and size and so it is proper for use in fuel cell hybrid vehicles [7]. Also, the proper rating between these two elements has an important impact on system performance.

3.3. Application of hybrid storage

There are numerous applications of battery and ultra-capacitor dual storage system seen in literature, which mainly include early studies and they more or less focused on the feasibility and implementation considerations [8]. There exist different configurations of multiple sources used in vehicle. For example, use of three sources composes of fuel cells, ultra-capacitor and solar cells [9] or multiple sources based on combustion engines, battery and photo-voltaic source [10].

Hybrid storage systems typically consist of sources with different time constants. Accordingly, it can be interpreted that in a hybrid vehicle, the prime mover provide DC component of power while buffer is responsible for providing the fluctuations around the mean value, Hence it prevent the working points of main source to move away from its optimal range [3]. In Ref. [11] energy management in a fuel cell vehicle with hybrid storage (batteries and ultra-capacitors) performed using fuzzy logic.

Unfortunately, the optimal control problem in hybrid vehicles with more than two energy sources is not solved and the existing work in this area is usually based on heuristic structures.

In most studies on hybrid vehicles with hybrid storage, the same approach as with simple storage is used, in other words, in the storage assembly, battery plays role of primary source and the ultra-capacitor plays the role of an auxiliary buffer. With this attitude, the problem of energy management deals with only one degree of freedom in power distribution and firstly, the supervisory control is designed independently based on primary source and battery and then a fraction of battery power allocated to ultra-capacitor based on a local controller. For example to control power between three energy sources, Yu presented a two-level control strategy [8]. He used a primary controller to distribute power between fuel cells and batteries and a secondary controller that ultra-capacitor help the battery.

Unlike the above approach, in the present study, battery and ultra-capacitor considered independently and each has its own degree of freedom in the power distribution. Fortunately this approach has been approved in Ref. [12]. This reference acknowledged that without changing control strategy of primary power source, the use of ultra-capacitor source cannot provide significant improvement in fuel consumption. This reference has provided rules for the optimal power management by studying the role of an ultra-capacitor in fuel consumption of a fuel cell hybrid vehicle, upon which the battery is responsible for fuel cell power management and Ultra-capacitor is responsible for battery power management. Based on the results of this reference, the control strategy has an effective role in performance and sizing of system. Of course this article states that the appropriate control strategy has a great impact on the sizing of components, but its effect on system performance (acceleration, fuel consumption) is lower.

In Ref. [13] the optimal solution of energy management (with two controls and three states) has been evaluated in several driving cycles, and the potential to reduce fuel consumption in these cycles has been investigated. Due to the large amount of calculations, this problem is reduced to three simpler problems and solved using dynamic programming. The structure of this hybrid vehicle consists of a conventional combustion engine and a flywheel as an energy buffer.

Kum [5] presents one of the few works that used MDDP with continuous variables. In this reference the cost function include fuel consumption and emissions. In this paper, in addition to the battery state of charge which is usually used in energy management studies to include battery dynamic, the catalyst temperature is considered to include the thermal dynamics of the catalyst. This study developed a systematic method to design the cold-start supervisory algorithm. Then some rules are extracted for real-time applications, which accelerating the warming of catalyst tends to reduction in pollution of engine when working in cold conditions. Also a two-mode hybrid electric vehicle that has two state variables (battery state of charge and speed of combustion engine) considered on a combined series-parallel arrangement [14].

3.4. Dynamic programming

In recent years the dynamic programming is used to solve various dynamic optimization problems in vehicle such as determining the optimum strategy of gear shifting [15], determine the optimal topology of driveline [16], design of power split device [17], and reduction of trip time [18]. One of the systematic applications of dynamic programming (single state single input) is solving optimal energy management problem in hybrid vehicles. Of course some multi-dimensional problems solved with single control single state (SCSS) dynamic programming with help of special tricks. For

example Ngo has been used dynamic programming to control gear shifting in a parallel hybrid vehicle [19]. This type of problem belongs to integer programming and includes two state variables (gear position & engine on-off condition) and one control variable (gear shift command). Romaus considered the optimal energy management problem of an electric car with hybrid storage (batteries and ultra-capacitors) and solved it by using stochastic dynamic programming and claims that this combination severely reduces storage losses [20].

3.5. Computational cost

Solving multi-input multi-state energy management problems by dynamic programming is theoretically possible, but practically is very difficult due to high computational cost, and unfortunately few cases have been reported in the literature. In this regard DP can be used to solve long missions only with small amount of variables [4]. Perez reported that computational cost is a deterrent issue for problems with more than one dimension and points out that by adding an additional state (such as addition of ultra-capacitor bank), the computational cost becomes a major obstacle [6].

Some of the works such as Ref. [21] is mixed integer programming problem that despite being a multi-input multi-state, the additional variables are of integer type (the [22] refers to ways for converting these problems).

Musardo [23] mentioned several hours of computing time to solve the optimal control in a cycle (such as FUDS) by using dynamic programming on a Pentium IV. Also Kang [24] referred to time about one month for simulation of a problem with two state variables in the NEDC cycle and on the Pentium II. The energy management problem with dual storage buffer has been considered in Ref. [22]. This reference acknowledged that the problem cannot be solved in limited time when there exist two state variables. Referring to the shortcomings of dynamic programming, Åsbogård [25] talks about little hope for considering more dynamics (such as battery life and engine thermal dynamics).

However all of these cases acknowledged the high computational cost of MDDP, which tends to implementation difficulties.

4. Vehicle arrangement

A general high level model as shown in Fig. 1 is used and related elements are connected to each other based on power flow. To connect these systems, power electronic equipment such as power converters and inverters are required [26], but the implementation details of the sources ignored due to variety of configurations and hardware complexity. This research used an FCHV comprises of fuel-cells and a combined battery and ultracapacitor accumulator. The electric energy produced by these sources drive an electric motor that in turn propel the car. The required data for a case study (proportional to a medium-size passenger sedan) are collected from authorized references.

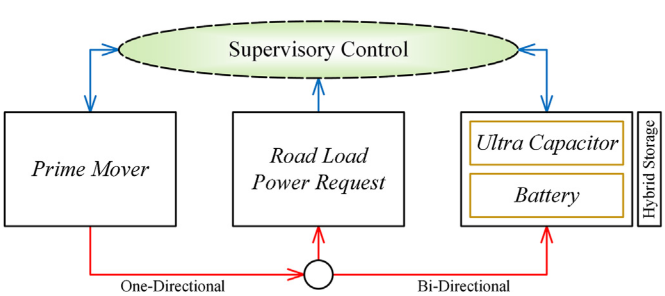

Fig. 1. Schematic arrangement of powertrain.

The aim of a hybrid control strategy is how to instantaneously distribute the power on primary energy source and electrical accumulator to satisfy the road load power requirement while meeting the instantaneous and global constraints and objectives. The energy-balance equation is the starting point of each energy management study, which the different sources contribute to provide demand power:

$$\underbrace{\underbrace{P_{fc}}_{\text{Fuel-Cells}}}_{\text{Primary Source}} - \underbrace{\underbrace{P_{dc}}_{\text{Electric Motor}}}_{\text{Road Load Demand}} + \underbrace{\underbrace{P_{bt}}_{\text{Battery}} + \underbrace{P_{uc}}_{\text{Ultra-Capacitor}}}_{\text{Hybrid Accumulator}} = 0 \tag{1}$$

5. Modeling

This section briefly describes models of hybrid vehicle subsystems based on existing rich literature. Guzzella [27] has systematically surveyed different modeling approaches for vehicle longitudinal dynamic and powertrain in forward dynamic and inverse quasi-static form. In addition, he has presented a modeling tool for powertrain of hybrid vehicles using quasi-static approach.

The models used in energy management applications normally have lumped and time invariant parameters and they are represented mathematically in constant coefficient differential-algebraic equation (DAE) form. Sometime these parameters depend on system state variables, which in quasi-static models, the look-up table and experimental data fitting are used to describe the component's complexity. Each look-up table is a nonlinear mapping between operational variables and system parameters. These mappings are measured at steady-state conditions so their transient behaviors ignored.

In this research only the dynamic of storage buffer has been considered so the other components were modeled in quasi-static manner and based on power split approach. So the components modeled with respect to efficiency and compatible with power flow requirements.

5.1. Vehicle mechanical dynamics

5.1.1. Driving cycle

Driving cycle is a standard velocity profile originated from the realistic driving pattern and one of its applications is evaluation and certification of automotive fuel consumption and emissions. Each driving cycle used to simulate a specific driving condition, for instance, UDDS represents urban driving for lightweight vehicles. In researches related to fuel consumption, vehicle is driven along a driving cycle which considered as a priori known disturbance to energy management problem.

The standard driving cycle has an important role on optimal energy management result in hybrid vehicles. For example, supervisory control in a short driving cycle with low power request and no charge sustenance restriction, converges to trivial solution that all required power supplied only by the buffer source. Of course because of losses in vehicle and its components, charge sustaining constraint always forces primary power source to deliver some fraction of power (at least equal to lost power).

In this research, five driving cycles have been used in simulation to reflect different operating conditions of vehicle, including ECE-15 (low speed urban run), NYCC (low speed low run), EUDC (urban running), SFTP (high speed high acceleration running) and NEDC (dynamometer test Emissions evaluation).

5.1.2. Vehicle longitudinal dynamics

Among the direct or inverse vehicle dynamic modeling, in most of the researches on energy management, the inverse approach is

used. In inverse model, velocity and acceleration are inputs, while these quantities considered as state in forward models and so it needs to driver model to follow driving cycle.

In this research, the inverse model of longitudinal vehicle dynamic is used and its data represented in Table 1. By using a priori known driving cycle and solving inverse vehicle dynamic, the corresponding road power is calculated and after passing through driveline, the power demand from powertrain is determined [28]. This demand power must be corrected, because it might be out of component capacity. For instance, the accelerating power might be more than the power of both primary and buffer sources, or the braking power may exceeds the amount that can be absorbed by storage element.

This model is constructed based on the balance of forces acts in longitudinal direction [2]. The applied forces acting on vehicle are composed of two components: tire traction forces (originated from powertrain and brake) and resistance forces (road load) which their difference tends vehicle to accelerate. Vehicle resistance forces consist of aerodynamic drag, rolling resistance and uphill driving force that in general are function of velocity.

$$
\begin{aligned}
\sum F_x &= \underbrace{\sum F_t}_{\text{Traction}} - \underbrace{\sum F_r}_{\text{Road Resistance}} = \underbrace{m\dot{v}}_{\text{Inertial}} \\
&\& \\
F_r(v) &= F_r + F_a + F_g \\
&= \mu_r mg\mathrm{Cos}(\phi) + 0.5\rho_a C_d A_f v^2 + mg\mathrm{Sin}(\phi) \\
&\Rightarrow \\
P_t &= F_t v = F_r(v)v + m\dot{v}v
\end{aligned}
\tag{2}
$$

The P_t is tractive power satisfying conditions of vehicle motion.

5.1.3. Regenerative brake

In hybrid vehicles, some fraction of the braking energy (wasted in conventional vehicles) can be recovered by operating the motor drive as a generator and restoring it into the power storage [2]. Of course the regeneration process depends on velocity and deceleration, structure of brake and generator, and ability of battery in energy absorption. Before passing braking power through the generator, a constant efficiency (50%) is used for modeling regenerative braking [29]. In heavy braking conditions, auxiliary mechanical braking system is required. In this research, mechanical braking has not modeled.

5.1.4. Axle & transmission

After the power produced by powertrain passes through electric machine, delivered as mechanical power to driveline and wheels (and vice versa). The collection of driveline components (includes motor transmission, final drive and wheels) which are located after electric machine, modeled as a static ratio.

$$
\begin{Bmatrix} \omega_e \\ T_e \end{Bmatrix} = \underbrace{\begin{bmatrix} N_r & 0 \\ 0 & N_r^{-1} \end{bmatrix}}_{\text{Reduction Gear}} \underbrace{\begin{bmatrix} N_d & 0 \\ 0 & N_d^{-1} \end{bmatrix}}_{\text{Differential}} \underbrace{\begin{bmatrix} R_w^{-1} & 0 \\ 0 & R_w \end{bmatrix}}_{\text{Wheel}} \begin{Bmatrix} V \\ F_t \end{Bmatrix} \tag{3}
$$

The above equation relates flow and effort components of power between electric machine and driving cycle, which is fed to machine model to determine demand power from powertrain. The general parameters of axle represented in Table 2.

Table 1
General vehicle data.

SAMAND (Iran National Car from IKCo Brochure)		
Total mass	m	1080 kg
Frontal area	A_f	2 m^2
Drag coefficient	C_d	0.51
Rolling resistance	μ_r	0.009
Air density	ρ_{air}	1.202 kg/m^3

5.2. Electric machine

There is a wide range of electric machines, which [2] has presented a general classification. Traction motors such as Standard DC, Induction AC and Brushless DC with a maximum power range of 20–50 kW, are used in hybrid-electric vehicles. Due to complexity of these systems, a wide range of models has been presented in literature. In most references, steady state motor model is used as an initial effort for control applications, and the motor considered as a static block associated with power losses [27]. These power losses are presented as algebraic relations or lookup tables. In this paper, a quasi-static model has been used based on the efficiency map shown in Fig. 2 [28]. Other parameters represented in Table 3 (adapted from ADVISOR^{TM} Library).

$$
\eta_m = \eta_m(\omega, T_m) \tag{4}
$$

Model inputs are angular velocity and requested torque from tire and output is the power necessary to produce requested torque.

$$
\eta_m = \frac{P_{Out}}{P_{In}} = \frac{T_m \omega}{vi} \tag{5}
$$

5.3. Storage buffer

Buffer is a key element in energy management which improves stability, reliability and quality of primary power sources. In most of energy management studies, buffer elements are the only considered dynamic, so the modeling of these elements discussed in more details. Sometimes it is exaggerated that battery is energy reservoir while ultra-capacitor is power reservoir. As a rule of thumb, ultracapacitor supplies power more efficiently than batteries, assuming equal power density [12]. Due to similar application, there are great similarities in modeling of these elements. Here a common template based on equivalent circuit is used to model these two components, which has the same structure and similar inputs and state variables and provide sufficient accuracy for design study and control applications.

In great deals of initial estimate to energy management problems, the dynamic of storage considered simply as an integrator of input while terminal power and state of charge respectively considered as model input and state. Nonetheless for energy management purposes between these two elements, the time constant and losses should involve in the model because various integrators treated as an equivalent integrator by solver.

5.3.1. State of remainder

In order to show the amount of supply in the storage, the state of charge quantity is used, which present the correlation between existing charge and storage capacity. This quantity is an important operational indicator in hybrid vehicles and the performance of storage system take its maximum in the middle of this interval (0.4–0.6). Theoretically SOC is a normalized dimensionless unit (between 0 and 1), but in practice an upper

Table 2
Parameters of axle (wheel and gearbox).

SAMAND (Iran National Car from IKCo Brochure)		
Wheel radius	R_w	0.33 m
Final drive ratio	N_d	4.07
Reduction gear ratio	N_r	1.6

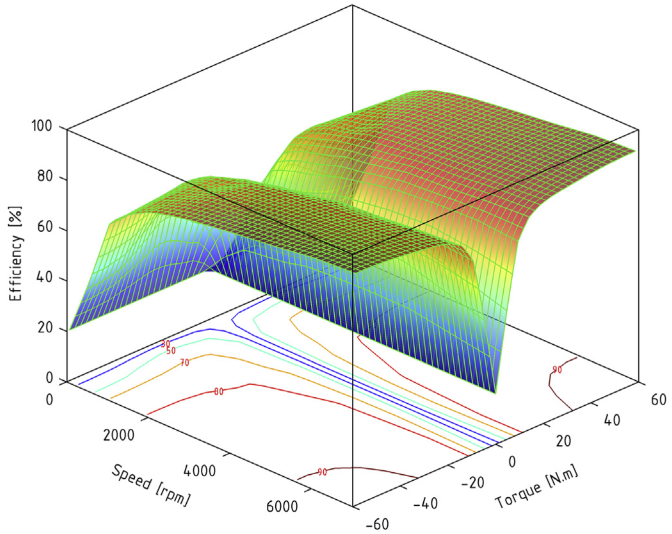

Fig. 2. Spatial map of electric machine efficiency.

bound (provide a margin for safety) and a lower bound (to overcome power-conversion Inefficiency of buck-boost converter in extremely low voltage conditions) is defined as practical limits [30].

$$\mathrm{SOC} \in [S_{\min},\ S_{\max}] \tag{6}$$

5.3.2. Charge sustenance

Charge sustenance means the net energy consumption in the buffer in a complete driving cycle is zero, that is, the energy level in the storage at the beginning and the end of the driving cycle is the same. In contrary "charge depletion" means maximum battery usage (plug-in vehicle). The buffer charge sustaining constraint should be included in optimal power management problem. It does provide the ability to benchmark and compare various controllers, in other word the storage does not make any contribution in vehicle power supply, but it only serves as an intermediate device for saving and loading of energy. To compare a real-time controller with the optimal solution, the final value of state of charge obtained from the optimal solution should be replaced with final value of state of charge in a real-time controller. In fact, little changes in the final state of charge value may lead to widespread use of battery.

Of course, in the real-time controllers this constraint is considered too to avoid the frequent charge and discharge of battery. In most of the studies related with real-time control topic, state of charge has been controlled within specific range. In some cases like [11] a reference signal has been used, which has a maximum (stopped vehicle has no ability to store regenerative braking energy) and minimum (vehicle driving at maximum speed has no chance to receive more energy to accelerate) limits.

5.3.3. Storage dynamic

The equivalent circuits of hybrid storage elements are shown in Fig. 3 which has many features in common. Based on "motoric references", positive terminal power is corresponding to buffer charging. Since the terminal voltage is positive, the direction of current depends on the sign of terminal power.

Based on the existing idealized models, charge process of these two elements is completely different. Charging the ultra-capacitor, increase its voltage until the voltage balances with external voltage and current flow diminished, while in the battery charging process, current and so on charge level continues permanently. So if the battery state of charge crosses the allowable boundary, its charging process should be stopped.

The general form of storage dynamic with respect to state of charge definition is as follows

$$\dot{S} = \Theta(S, P)$$
$$\left\{ S = \frac{\text{Remaining Charge}}{\text{Capacity}} \Rightarrow \dot{S} = \frac{i(t)}{Q_{\max}} = \frac{P(t)}{Q_{\max} V(t)} \right\} \tag{7}$$

Since the terminal power is input of storage model, the current (or voltage) of terminal should be calculated based on it. Looking at Fig. 3 and using the Kirchhoff's law (KVL), the circuit current is determined.

$$v_t - i_t R_s - v_c = 0 \Rightarrow \underbrace{P_t}_{\text{Terminal}} - \underbrace{i_t^2 R_s}_{\text{Series Loss}} - \underbrace{i_t v_c}_{\text{Storage Element}} = 0 \tag{8}$$

The above quadratic equation which is applicable to both battery and ultra-capacitor elements, has two solutions for current as follows:

$$i = \frac{-v_{oc} \pm \sqrt{v_{oc}^2 + 4R_s P_t}}{2R_s} \Rightarrow i_t = -\frac{v_c}{2R_s} \pm \sqrt{\left(\frac{v_c}{2R_s}\right)^2 + \frac{P_t}{R_s}} \tag{9}$$

where, v_c is the voltage of storage element. By determining fraction of terminal current allocated for energy storage element and rewriting in terms of state of charge, the dynamic of system is determined.

5.4. Battery

Because of complexity in chemical reactions of battery, it is difficult to develop a model based on its physical phenomena that can predict transient behavior.

In this paper, the simple model of equivalent RC circuit is used as shown in Fig. 3, which consists of an ideal voltage source (open circuit voltage) and an internal resistance, and characteristic curve of this two element for a simple cell shown in Fig. 4 [31]. Since the internal resistance depends on current direction, there are two different curves for charge and discharge situation.

Based on Table 4 [20], the capacity of battery bank is as follows:

$$E = (N_s V_r)(N_p Q_{ah}) = 4 \times 12 \times 26/1000 = 1.248\ \text{kWh} \tag{10}$$

5.4.1. Equation of model

To determine battery dynamic, the circuit current from equation (9) substituted in state of charge equation.

$$Q\dot{S}_b = -\frac{V_{oc}}{2R_s} \pm \sqrt{\left(\frac{V_{oc}}{2R_s}\right)^2 + \frac{P_t}{R_s}}$$
$$\left\{ Q = N_p \widehat{Q},\ V_{oc} = N_s \widehat{V}_{oc},\ R_s = \frac{N_s}{N_p} \widehat{R}_s \right\} \tag{11}$$

The above equation is a battery dynamic model in the general form of relationship (7). Since the state variable does not in the right hand side of the equation, battery dynamic is a simple integrator.

Table 3
Parameters of electric machine.

Permanent magnet (PMS)		
Nominal power	P_n	32 kW
Efficiency bound	η_m	20–92%

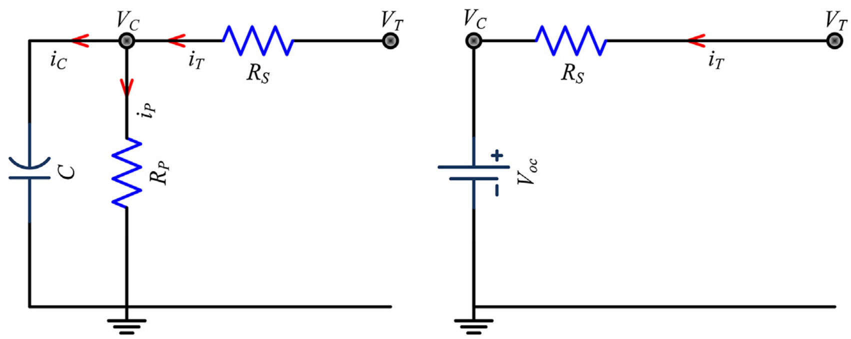

Fig. 3. Equivalent circuit of battery and ultra-capacitor.

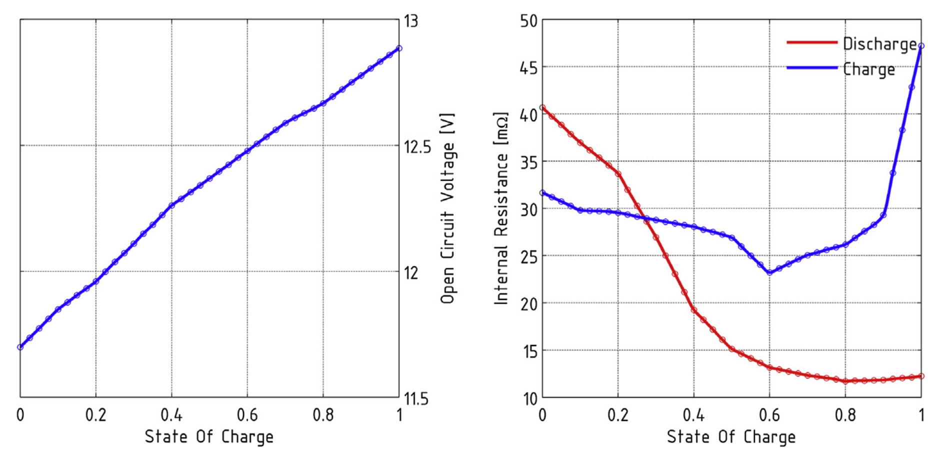

Fig. 4. Variation of open circuit voltage and internal resistance as a function of charge level.

5.5. Ultra-capacitor

In most of the researches on power management, ultra-capacitor modeled using equivalent circuit and normally a first-order [32] or second-order [33] models. Here an equivalent circuit model presented in Fig. 3 used which consists of a capacitor, a series resistance (resistance against the charge and discharge) and a parallel resistance (overall leakage).

5.5.1. Equation of model

In order to determine the dynamic equation of model, first KCL was applied on connecting node of circuit elements and then substitute terminal current from equation (9).

$$
\begin{aligned}
& i_{\mathrm{c}} + i_{\mathrm{p}} = i_{\mathrm{t}} \quad \Rightarrow C\tfrac{\mathrm{d}}{\mathrm{d}t} v_{\mathrm{c}} + \tfrac{v_{\mathrm{c}}}{R_{\mathrm{p}}} = i_{\mathrm{t}} \\
& \Rightarrow C\tfrac{\mathrm{d}}{\mathrm{d}t} v_{\mathrm{c}} + \tfrac{v_{\mathrm{c}}}{R_{\mathrm{p}}} = -\tfrac{v_{\mathrm{c}}}{2R_{\mathrm{s}}} \pm \sqrt{\left(\tfrac{v_{\mathrm{c}}}{2R_{\mathrm{s}}}\right)^2 + \tfrac{P_{\mathrm{t}}}{R_{\mathrm{s}}}}
\end{aligned}
\tag{12}
$$

By assuming constant capacitor coefficient, the state variable (state of charge) is as follow:

Table 4
Battery parameters.

Valve-regulated lead acid battery		
Nominal voltage	V_{r}	12 V
Coulombic efficiency	η	90%
Charge capacity	Q_{ah}	26 Ah
Internal resistance (average)	$\overline{R}_{\mathrm{chg}}$, $\overline{R}_{\mathrm{dis}}$	28.9, 20.9 mΩ
Bank	N_{s}, N_{p}	4, 1

$$
S_{\mathrm{c}} = \frac{Q_{\mathrm{c}}}{Q_{\mathrm{max}}} = \frac{Cv_{\mathrm{c}}}{Cv_{\mathrm{max}}} \Rightarrow v_{\mathrm{c}} = S_{\mathrm{c}} v_{\mathrm{max}} \tag{13}
$$

The dynamic equation of ultra-capacitors obtained by combining equations (12) and (13), which is in the general form of equation (7). This equation represents a first-order nonlinear system.

$$
\begin{aligned}
& C\dot{S}_{\mathrm{c}} = -\left(\tfrac{1}{2R_{\mathrm{s}}} + \tfrac{1}{R_{\mathrm{p}}}\right) S_{\mathrm{c}} \pm \tfrac{1}{2R_{\mathrm{s}}} \sqrt{S_{\mathrm{c}}^2 + \tfrac{4R_{\mathrm{s}}}{V_{\mathrm{max}}^2} P_{\mathrm{t}}} \\
& \left\{ C = \tfrac{N_{\mathrm{p}}}{N_{\mathrm{s}}} \widehat{C}, \;\; R_{\mathrm{s}} = \tfrac{N_{\mathrm{s}}}{N_{\mathrm{p}}} \widehat{R}_{\mathrm{s}}, \;\; \widetilde{R}_{\mathrm{p}} = \tfrac{N_{\mathrm{s}}}{N_{\mathrm{p}}} R_{\mathrm{p}} \right\}
\end{aligned}
\tag{14}
$$

Based on the data represented in Table 5 [34], the capacity of ultra-capacitor bank is as follows:

$$
E = \frac{1}{2}\left(\frac{N_{\mathrm{p}}}{N_{\mathrm{s}}} C\right)(N_{\mathrm{s}} V_{\mathrm{r}})^2 = \frac{0.5 \times (2/9) \times 500 \times (9 \times 16)^2}{1000 \times 3600} = 0.32\,\mathrm{kWh} \tag{15}
$$

5.6. Fuel cell

Fuel cells have various types which normally categorized in terms of their electrolyte. One of these types that widely used in vehicle propulsion system is Polymer-Electrolyte Proton Exchange Membrane (PEM) [35]. According to multi-disciplinary nature of fuel cell, there are different modeling approaches in literature with different level of complexity. It is common to use quasi-static models of fuel cell in energy management application and it assumed to be well-regulated, to operate in nominal conditions.

Table 5
Ultra-capacitor parameters.

Maxwell BMOD0500B01		
Capacitance	C	500 F
Rated voltage	V_r	16 V
Internal resistance	R_s	2.1 mΩ
Leakage current	I_l	5.2 mA
Bank	N_s, N_p	9, 2

To avoid complex thermodynamic and electrochemistry equations, an efficiency-based model used for fuel cell, which can describe common types of fuel cells and its goal is to calculate fuel consumption from requested power (the inverse of model shown in Fig. 5). For sake of simplicity, the internal phenomena like activation, mass transport over potential and internal resistance of fuel cell were ignored. There is no bound considered on stack variables like current, voltage and temperature.

5.6.1. Fuel consumption

In initial estimates, the fuel cell efficiency is defined based on Gibbs free energy, Helmholtz function and enthalpy [35], which depends on fuel cell reactions type. According to fuel cell structure shown in Fig. 5, the fuel processing and its irreversibly affect efficiency and also a fraction of power produced supplied to auxiliaries equipment mostly related to the air compressor. Here the fuel cell data adapted from ANL50H model of Advisor library. Having efficiency curve of fuel cell shown in Fig. 6, consumption rate and its specific fuel consumption determined [30].

$$\dot{m}_{H_2} = \frac{P_{fc}}{\eta\left(P_{fc}\right) \times LHV_{H_2}} \tag{16}$$

Here fuel consumption measured in unit of equivalent gasoline liter per 100 km. If vehicle traveled x meters along a driving cycle and consumed y grams of hydrogen, the gasoline equivalent consumption based on LHV is as follows [36]:

$$FC = 378.54(y/x)[\text{Gasoline Equivalent L}/100\text{ km}] \tag{17}$$

5.7. Final model

By assembling all of described sub models and implemented in MATLAB, the final is derived in an analytical viewpoint, the nonlinear state space representation of final model is as follows:

$$\dot{\vec{x}}(t) = F(\vec{x}(t), \vec{u}(t), w(t)) \tag{18}$$

Since there are two storage elements in this research, system dynamic model consist of two state variables (x) corresponding to battery and ultra-capacitor charge and two control inputs (u) are the fuel cell and ultra-capacitor power. The driving cycle considered as a priori known exogenous variable (w).

The number of control variables can be reduced by augmenting a sample value to control array and thereupon removing one dimension of the problem. For instance, the (-1) can be used to represent primary source on/off state. This is suitable in such cases that control variables are dependent on each other (i.e. the input to mechanical brake and primary power source do not activated simultaneously), and they should be considered in a multi-dimensional framework if they were independent.

6. Optimal supervisory control

The supervisory control is a top-level structure that provides control signals required for system resources. This request feeds as a reference signal to low-level controllers and it would be provided by ideal regulators and proper servo-loops.

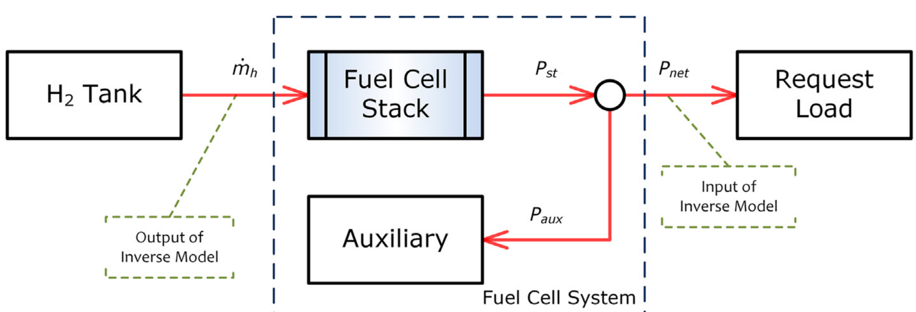

Fig. 5. Structure of fuel cell (forward model).

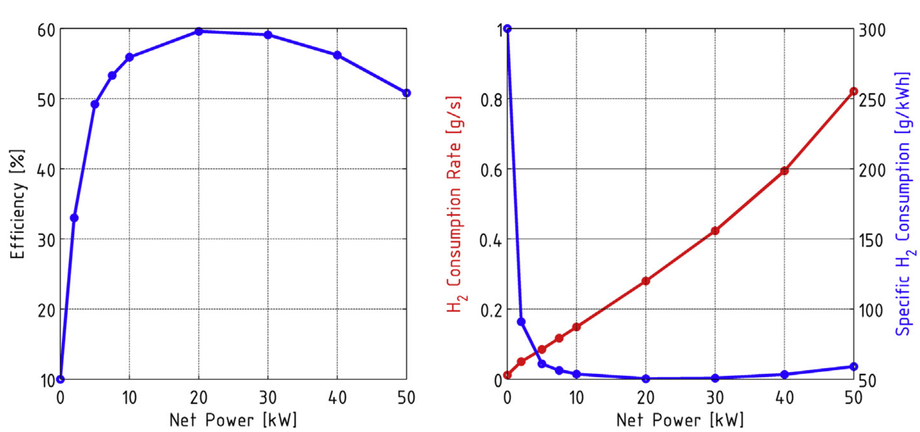

Fig. 6. Characteristic curve of fuel cell.

In this section, firstly the optimal supervisory control of energy management is formulated with respect to the final vehicle model and then solved using dynamic programming. Fig. 7 illustrates the overall structure of problem. Finally, the problem was simulated in various driving cycles, and potential of utilizing hybrid storage on the vehicle performance is investigated.

6.1. Formulation of dynamic optimization

In most of simple models used in the offline management of energy in hybrid vehicles, the storage charge level is the only state of system. Generally the dynamic which needs to be controlled or affects the cost function and constraints should be considered as state variable in dynamic programming. Because of the charge sustaining constraint in hybrid vehicles, the storage charge level will be considered as state variable in dynamic programming. Note that if the charge sustaining constraint omitted and no boundary limits on state of charge then there is no need to include storage dynamics.

In this paper, a vectorized-high performance dynamic programming code is used, but the powertrain model should be simplified enough, because the computational cost associated with multi-control multi-state is still a deterrent factor in the application of dynamic programming.

6.1.1. General formulation of optimal control

The profile of optimal control problem studied in this paper are as follow: constant time interval, multi-state multi-control, constant initial and desired final state, fixed definition interval of state and control variables, a priori known disturbance. These characteristics can be summarized in following formulation:

$$\begin{array}{ll}
\text{Cost Functional} & J = \zeta\left(x\left(t_f\right)\right) + \int_{t_0}^{t_f} \xi(x(\tau), u(\tau), \tau)\mathrm{d}\tau \\
\text{Optimal Control} & u^*(t) = \underset{u(t)}{\operatorname{argmin}} J(x_0, u(t)) \\
\text{System Dynamics} & \{\dot{x}(t) = F(x(t), u(t), t) \\
\text{Boundary Condition} & \begin{cases} x(t_0) = x_0 \\ \overline{x}\left(t_f\right) = x_f \end{cases} \\
\text{Admissible Set} & \begin{cases} x(t) \in \mathscr{X}(t) \subset \mathbb{R}^n \\ u(t) \in \mathscr{U}(t) \subset \mathbb{R}^m \end{cases}
\end{array} \tag{19}$$

6.1.2. Cost components

In vehicle design consideration, there should be a harmony between drivability and fuel-related (consumption and pollution) criteria and no one sacrificed for other. Of course the weighting factor for mentioned indicators depends on the vehicle class. Therefore there is more attention to fuel considerations in economical cars which mostly work in the traffic cycles, while in the sports car, performance is weighted more. In this research, based on software implementation considerations, the cost function divided into three components: boundary component (charge sustenance), instantaneous component (fuel consumption), and rate component (frequency of fuel cell operation).

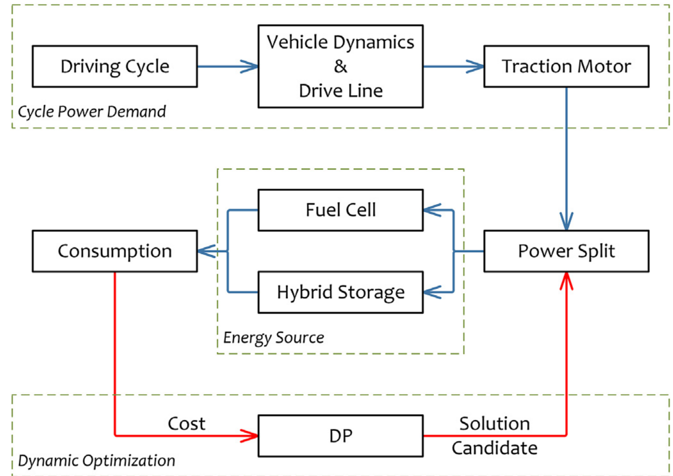

Fig. 7. Structure of dynamic optimization.

The satisfaction of charge sustenance is a big challenge in energy management problem, and here it is implemented as a "soft" constraint, i.e. a penalty added to cost function based on charge difference from desired final value. Also another component added to cost to decrease frequency of fuel cell operation (reduce frequent fuel cell turning on and off) [37]. The implementation of this term is complicated because it requires knowledge of input at both side of each step.

The main component of the cost function is fuel consumption of primary source (hydrogen). So it is clear that by utilizing the integrator model, the storage element has higher priority in energy delivering, unless the demand power exceed the maximum storage capability or the state of charge reaches to its limitation boundaries, which the main power source comes to share.

6.2. Dynamic programming

Dynamic programming is a powerful method for the numerical solution of dynamic optimization problems [6]. Optimal control is a part of this family and the use of dynamic programming to solve these kinds of problems has been widely studied in the literature [38]. This method can be applied to deterministic as well as stochastic problems and simply deal with nonlinear dynamics and constraints and guarantees achievement of global optimality (provided the solution exists). In this method, firstly the problem must be discretized (partitioning time, state and control variables as in Fig. 8). So there is only finite amount of possible solutions. This algorithm only searched a part of the acceptable solution within all possible solution space using "Bellman principle of optimality".

In this section the formulation of dynamic programming summarized for solving optimal control problems based on the notation presented in Ref. [39]. See Ref. [27] for theoretical principles and formulation.

$$J(x_{bt}, x_{uc}, u_{fc}, u_{uc}) = \underbrace{\underbrace{w_{bt}\left(x_{bt}^N - x_{bt}^1\right)^2}_{\text{Battery}} + \underbrace{w_{uc}\left(x_{uc}^N - x_{uc}^1\right)^2}_{\text{Ultra Capacitor}}}_{\text{Charge Sustaining Penalty}} + \underbrace{\Delta t \sum_1^{N-1} \dot{m}_f(u_{fc})}_{\text{Fuel Cell Consumption}} + \underbrace{\lambda \sum_1^{N-1} \left(Sign\left(u_{fc}^k\right) - Sign\left(u_{fc}^{k+1}\right)\right)^2}_{\text{On/Off Frequency}} \tag{20}$$

The discrete formulation of dynamic programming is based on a recursive relationship with specified initial condition.

$$
\begin{array}{ll}
\forall x \in \mathcal{X}_N & \Rightarrow J_N(x_N) = \zeta(x_N) \\
\begin{cases} k = \{N-1, \cdots, 2, 1\} \\ \forall x \in \mathcal{X}_k \end{cases} & \Rightarrow J_k(x_k) = \min\limits_{u_k \in \mathcal{U}_k} \{\xi_k(x_k, u_k) + J_{k+1}(x_{k+1})\}
\end{array}
\tag{21}
$$

In above recursive relation, J_k is called “Cost to Go” map or value function, which for every point of state-time space (x_k) gives the cost to move on optimal trajectory from that point to final time. Also u_k is optimal control for the departure from this point to the next step on optimal trajectory.

This map is usually implemented as a lookup table. To determine the optimal trajectory corresponding to a specific point in state-time space, it is sufficient to determine the corresponding optimal control and then solving system dynamics to obtain next point on the optimal trajectory. Now repeating this process toward the final Time, the system trajectory and series of optimal control is determined.

6.2.1. Code-vectorized development

Although the dynamic programming heavily reduces the computational cost of determining the optimal solution in comparison to direct search, however, its application in energy management problems in long cycles and including multi-state multi-control variables is associated with difficulty. Guzzella presented a complete structure for definition of dynamic programming algorithm and standardized the definition of the problem, the data structure and required routine [39] and this paper will follow this conventions.

MATLAB package have optimal routines for matrices-development algorithms, which the significant improvement in performance obtained by eliminating iterative and loop structures. Because the energy management problem includes high amount of iterated matrix algebra, the vectorized formulation of the code will heavily improve its performance. Unfortunately there is no integrated works in this fields presented in literature. Among the existing works, only [37] noted to vectorization of dynamic programming code to improve the computational performance. Also in Ref. [27] the general format of vectorized pseudo-code of dynamic programming presented.

In conventional dynamic programming, three loops over time stage, state variables and control are required [27]. Using advantages of the code-vectorization, loops on the state variables and control can be eliminated. Elimination of these loops improved significantly computational performance, but need higher memory allocation. In fact, a loop consisting of a series of repetitive operations on elements of an array based on its index. In vectorized version, all operations on elements performed in one step.

7. Simulation and evaluation

In this study, the maximum potential of using hybrid storage on fuel consumption is determined relying on model-based approach. In this regard, firstly, the assembled model is discretized. Then using deterministic multi-dimensional dynamic programming, the optimal energy management solution corresponding to minimum fuel consumption is achieved.

To demonstrate the applicability of developed tool, three vehicle arrangements used including “No Storage”, “Single Storage” and “Hybrid Storage”. The first layout is similar to a conventional combustion vehicles except the internal combustion engine replaced by fuel cell power supply and since there is no storage system, the brake energy cannot recovered and the driving demand of cycle is fully propelled by fuel cell. The parameters of dynamic programming solver are represented in Table 6.

The convention of power flow used in following graphs presented as follow. Based on Fig. 1, the positive road power demand corresponds to power request from power supply elements (i.e. in

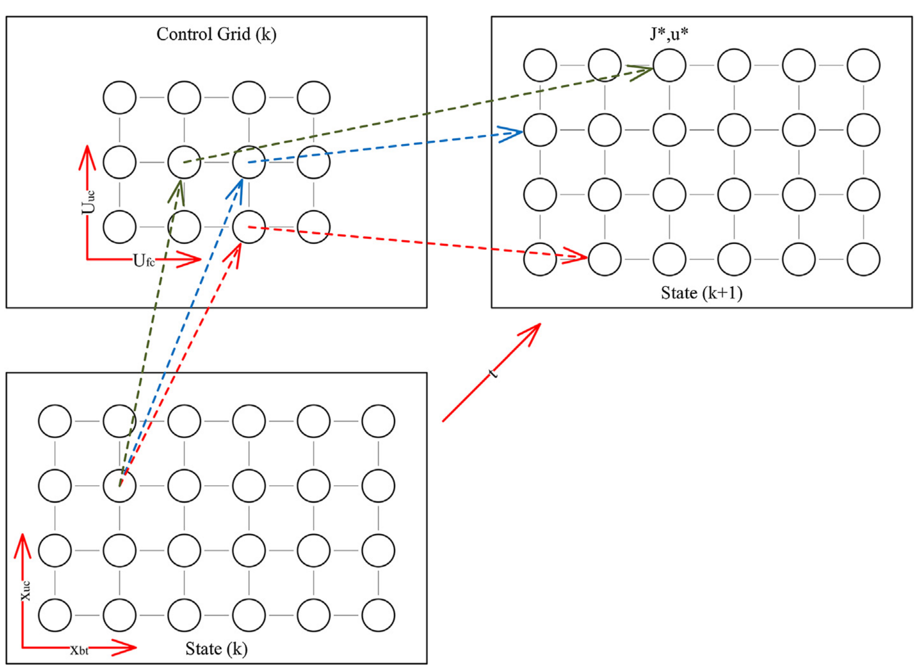

Fig. 8. The dynamic programming search pattern in each time stage.

Table 6
Parameters of dynamic programming.

Dynamic programming algorithm		
Single	N_x, N_u	81, 51
Multiple	$N_{x(bt,uc)}$	81, 41
	$N_{u(fc,uc)}$	51, 41
Interpolation	J, U	pchip, linear

accelerating or uphill climbing). Based on motoric references, positive buffer power corresponds to charging. By considering these two convention, the prime mover power (fuel cell) is positive only (deliver power). To properly present graphs, the negative of fuel cell power is utilized to avoid curves conflict.

The simulation results for the three specified vehicle arrangements presented in two parts: "time results" and "statistical results" as follow.

7.1. Time analysis of the optimal solution

Time results including time response of model variables presented for ECE-15 driving cycle, which used to demonstrate performance of developed tool and provide intuition understanding. Fig. 9 represents speed and acceleration (green stair-wise curve) of mentioned driving cycle. The horizontal bars on left show the speed frequency and density, indicating the concentration of specified driving cycle in different speed regions. Alongside with driving cycle, the time history of vehicle variables is presented in Figs. 10–12. In each arrangement, the colored-bands in driving cycle plot, used to distinguish between pure activity of primary (orange color in web version, pure hydrogen-propelled) and secondary (green color in web version, pure electric-propelled) source. In Fig. 10 the result of a fuel cell electric vehicle without storage buffer is shown. Since there is no recovery of braking energy, this vehicle has higher fuel consumption. In Figs. 11 and 12 the result of hybrid fuel cell vehicle is shown. The Power plot corresponds to optimal energy management control input, which shows power distribution between different power sources. The SOC plot reflects state variables corresponding to selected vehicle. As can be seen, the charge sustenance satisfied very well.

The histograms of fuel consumption vs. power are normally used to show characteristics of fuel cell operation [30,36]. According to Fig. 13, the optimal strategy tries to concentrate the density of the fuel cell operation around the optimum fuel consumption point (approximately 20 kW). In this figure, the mean and standard deviation (μ & σ) of fuel cell operating point is shown.

7.2. Statistical analysis of the optimal solution

Statistical results are a convenient way to compare the performance between single and hybrid storage, which the resultant fuel consumption for different configurations of vehicles has been presented for four driving cycles, NYCC, EUDC, SFTP and NEDC. A comparison of minimum fuel consumption (gasoline equivalent

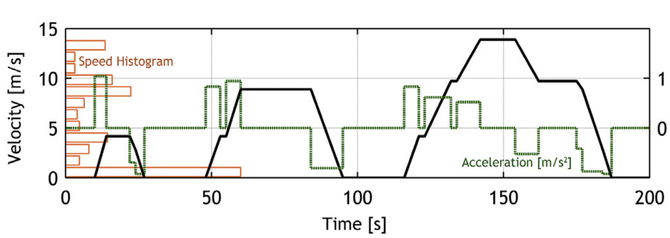

Fig. 9. ECE manual transmission city driving cycle.

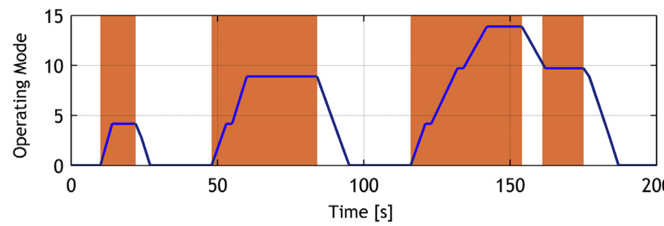

Fig. 10. Response of fuel-cell only vehicle.

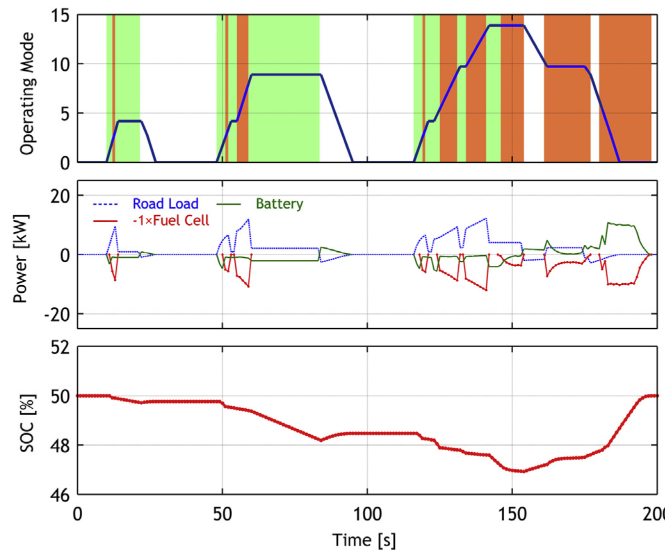

Fig. 11. Response of fuel-cell hybrid vehicle with single storage system.

liter per 100 km) between these two arrangements in different cycles is shown in Fig. 14.

Here are two rules to better justify the results presented in Fig. 14. *Rule 1*: The recovery of the braking energy is the main factor that distinguishes between the "No Buffer" layout and the other two layouts; as the latter could recover the braking energy using their storage buffer. As the figure shows, there is a significant improvement in the hybrid operation from the NYCC driving cycle

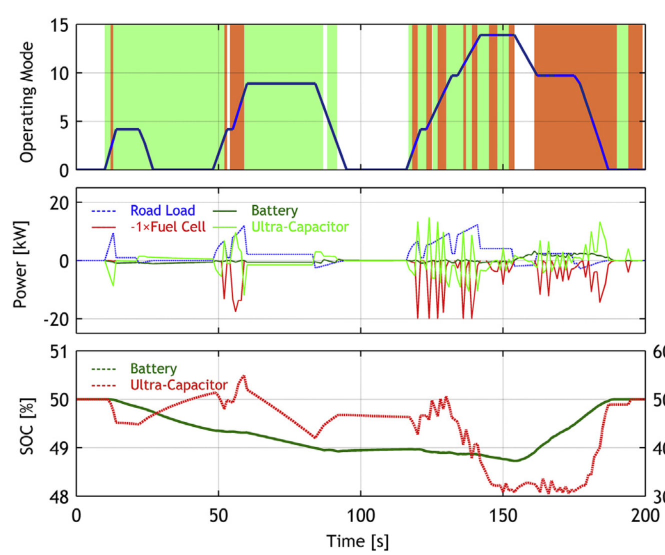

Fig. 12. Response of fuel-cell hybrid vehicle with hybrid storage system.

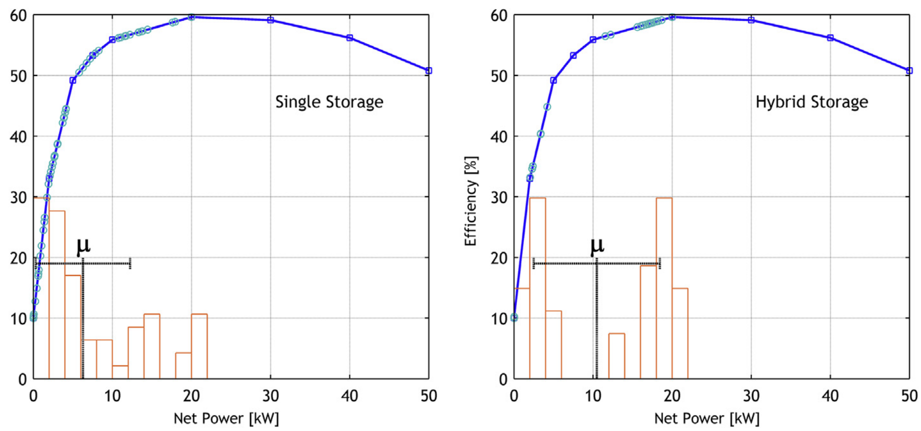

Fig. 13. Histogram of fuel-cell operation with single and hybrid storage system.

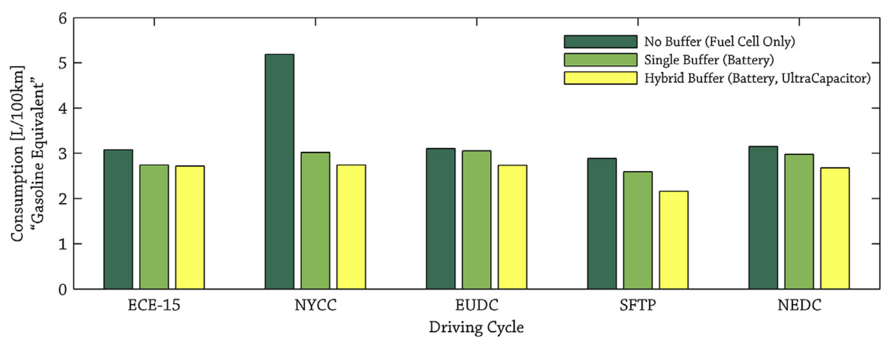

Fig. 14. Potential fuel consumption improvement in various driving cycles.

due to the high energy fraction of the braking. _Rule 2_: The ultracapacitor has a low energy content, so it could not contribute to the energy of the buffer, but since it has a high power absorption capability, it could help in the high power section of the driving cycle. As the figure shows, the ultracapacitor tends to somewhat decrease the fuel consumption in all cases. It could be seen that in the ECE-15 cycle which is a relatively smooth cycle, the ultracapacitor approximately made no difference; whereas in the EUDC, SFTP and NEDC cycles with more accelerations and decelerations, the ultracapacitor made more difference.

7.3. Validation of algorithm

Due to lack of experimental data, it is not possible to fully validate numerical accuracy of presented model, meanwhile the developed algorithm is evaluated by using different scenarios. To evaluate developed algorithm and assess numerical aspect, the scenario of removing the charge sustenance constraint is used. By removing this constraint and using integrator model (for storage elements) and assuming sufficient battery capacity, the optimal solution is focused on excess use of batteries. Another scenario that can be imagined is maximization of objective function. In this case, unlike the first scenario, the optimal solution is focused on use of fuel cell more than storage element. The results of simulation while removing charge sustaining certify the accuracy of general logic of simulation.

7.4. Computational load comparison

To demonstrate the performance of developed code, the computational cost associated with simulation of vehicle equipped with single and hybrid storage in two different cycles presented in Table 7. According to calculations time, it is clear that the computational load compared with those reported in the literature [23,24] is extremely lower. This proves the advantages of the code-vectorized implementation.

Characteristics of computer used in this research shown in Table 8. Due to functional structure overhead and graphical controls update, the pure computational load is less than amount reported.

Also be careful with grid partitioning of state and control variables, which demands for more memory request and for example the network with more than 200 divisions required about 6 GB of memory, and allocation of such amount of memory in 32-bit platform is a challenging issue.

Table 7
Computational cost comparison.

Cycle	Algorithm	Time (s)	Grid $X \times U$
ECE	Single	12	100×100
	Multiple	235	$(60,30) \times (60,30)$
NYCC	Single	28	100×100
	Multiple	712	$(60,30) \times (60,30)$

Table 8
Characteristics of computer software.

Hardware		Software	
CPU	Core2Quad 2.8 GHz	OS	Windows 7 X64
Memory	6 GB	MATLAB	2010A

8. Conclusion

The optimal energy management of a fuel-cell hybrid vehicle with hybrid storage investigated on a model-based approach. In this problem, a vehicle traveled along a priori known driving cycle. The objective function has two components: fuel consumption and charge sustenance. This problem has two state variables and two corresponding degrees of freedom in power distribution, so it was solved using multi-dimensional dynamic programming. The research goal is to determine the maximum potential of using hybrid storage in reducing fuel consumption.

The problem was simulated in different driving cycles to provide comparing the minimum fuel consumption between two arrangements: single storage and hybrid counterpart. The results indicate improvement in fuel consumption in presence of ultra-capacitor, because the deficiency of battery as a slow-storage is compensated by ultra-capacitor. The strong pulses of power could be absorbed by ultra-capacitor, so the hybrid storage has a large impact on acceleration performance and braking recovery.

The general format of optimal energy management solution in the described problem mainly depends on: dominant weight of fuel consumption in objective function, the point of maximum efficiency in fuel-cell performance curve, and characteristics of model used for storage elements. If simple integrator model is used for storage elements, then the nature of the optimal solution is of bang-bang type (fuel cell frequent on-off around optimal efficiency point). If storage lossy model is used, the optimal solution tries to make a balance between optimal operation of fuel cell, storage element and demand power, because in this situation the zero power relates to optimal operating point of storage while the fuel cell has low efficiency in low power region. The proper sizing of components has a great impact on performance of hybrid vehicle. This is well beyond the scope of this paper and it is suggested that the battery and ultra-capacitor be sized based on optimal energy management and represents the impact on fuel consumption. The satisfaction of charge sustenance constraint strongly affects the optimal solution and small deviations may tend to wide storage use. To provide a proper framework for design study and accurate comparison, the deviation from this constraint should be reduced. This is important in the present study, since it must be considered for both battery and ultra-capacitor simultaneously.

Due to lack of systematic study in context of optimal energy management solution in hybrid storage application, it is suggested that future studies concentrate on the application of algorithms such as iterative, stochastic and approximate dynamic programming and Pontryagin's principle, and the performance and computational cost is presented. Since there is no well-documented statistical information on manufacturing cost and hardware implementation, field studies providing such information in the field of hybrid storage systems, can help to investigate the balance between cost imposed on vehicle and improvement in vehicle fuel consumption.

The developed code and its user interface are placed freely in the address (sites.google.com/site/ansarey) in order to allow researchers use it for future development and generalize the software and its components library.

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